Karel HOLUB[1], Oldřich NOVOTNÝ[2]

PRELIMINARY RESULTS OF POLYNOMIAL AND RATIONAL APPROXIMATIONS OF SOME EMPIRICAL DEPENDENCES

PŘEDBĚŽNÉ VÝSLEDKY TESTŮ POLYNOMIÁLNÍCH A RACIONÁLNÍCH APPROXIMACÍ EMPIRICKÝCH ZÁVISLOSTÍ

Abstract

This paper deals with approximation of experimental seismic data obtained during field and laboratory seismic experiments. In particular, for obtaining best fitting curves, rational and polynomial approximations are tested.

Key words: approximation approaches, laboratory and field seismic experiments

Introduction

It is well known that field experiments and many laboratory measurements frequently lead to observed data that display a considerable scatter. Certain smoothing of such data is then usually required as the first step of their interpretation. For example, the classical Wiechert-Herglotz method of interpreting refraction measurements assumes the travel-time curve to be continuous with monotonous derivatives. Consequently, when applying this method, the observed data must be approximated by simpler functions of specific properties. This approach was successfully applied in many studies of the deep Earth's structure that were based on teleseismic observations.

However, the application of the Wiechert-Herglotz method to studies of the Earth's crust structure by deep seismic sounding met some problems. Namely, it was found that the real crustal structure in many regions differs significantly from the basic assumptions of the Wiechert-Herglotz method, i.e. a 1-D structure with continuous and monotonously increasing velocities with depth. Grad (1985) analyzed these problems in detail using synthetic data and he applied for approximation of travel-time curves polynomials of degree 4, 5 and 6. It was found that the Wiechert-Herglotz method reproduced well the structures, where a continuous increase in velocity with depth, boundaries of the second order and small velocity differentiation do exist.

Málek et al. (2004) interpreted refraction measurements along short profiles in the seismoactive region of western Bohemia where detailed geological maps were available. They recognized that within the individual geological units (such as plutons or crystallinicum) the vertical inhomogeneity usually dominates over lateral inhomogeneities. As opposed to rather pessimistic conclusions of Grad (1985), these new findings seemed to indicate that the Wiechert-Herglotz method could still be applied in some studies of the crustal structure if we restrict ourselves to smaller geological units and shallow depths only.

Similar smoothing problems occur in interpreting laboratory measurements. Compared with field experiments, laboratory measurements have the substantial advance in possibility to repeat some of experiments to receive the final approximation having a smaller scatter.

In the present paper some examples of approximation approaches using data from field experiments and laboratory tests are shortly discussed. For approximation of seismological data simple rational and/or polynomial functions are considered.

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[1] RNDr., DrSc., Institute of Geonics, v.v.i., AS CR, Studentská 1768, Ostrava, holub@ugn.cas.cz

[2] Doc. RNDr., CSc., Department of Geophysics, Faculty of Mathematics and Physics, Charles University, V Holešovičkách 2, Praha 8, on@karel.troja.mff.cuni.cz

Refraction profile Jakubčovice nad Odrou – seismic station Ostrava-Krásné Pole (OKC)

Travel-time curve approximation

The field experiments mostly used quarry blasts performed on the territory of northern Moravia and Silesia. These explosions were recorded at the temporary seismic stations, solitery distributed sites of observation and also along a short refraction profile (Holub et al., 2006). The last example is used here to test the approximation methods mentioned.

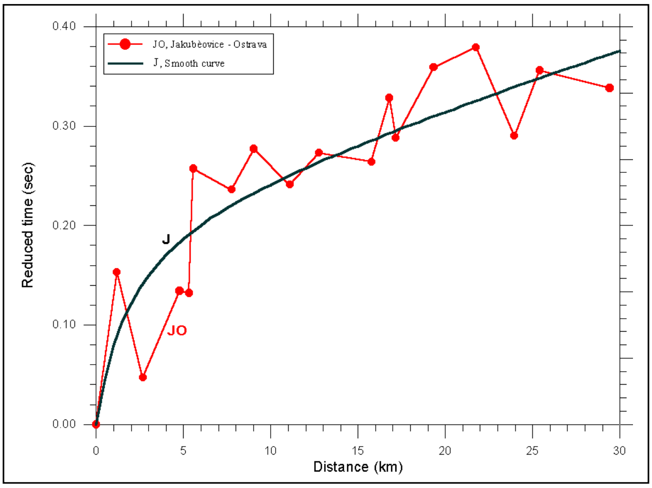

Fig.1 Reduced travel-time curve for the profile Jakubčovice nad Odrou-Ostrava (OKC) and the averaged smoothed curve denoted with the letter J (according to Holub et al., 2006a).

The length of the investigated refraction profile was approximately 30 km. The observed travel times of P waves, reduced with a reducing velocity of 6 km/s, are shown in Fig. 1 by isolated points (curve JO). One can see the obvious scatter of interpreted input data. The character of scatter is more pronounced due to scaling in reduced time.

As mentioned above, for application of the Wiechert-Herglotz method a smoothed travel time curve is needed. Therefore, were approximated the observed travel times by the rational function represented as a quotient of two polynomials given by the simple formula

$$t = \frac{a_1 r + a_2 r^2}{b_0 + r} \quad . \tag{1}$$

The unknown coefficients were determined by means of the conjugate gradient method. Taking into account the units used, i.e. epicentral distance r in km and time t in seconds, as the starting values of the coefficients we chose: a_1 = 0.3-0.6; a_2 = 0.017 and b_0 = 1. To reach new values of the coefficients, usually 10 iterations were sufficient. The best-fitting curve J in Fig. 1 is characterized by the parameters a_1 = 0.567, a_2 = 0.172 and b_0 = 2.00; the corresponding standard deviation is 0.046 s.

Application of the Wiechert-Herglotz method

Using refracted waves, the velocity-depth distribution in a vertically inhomogeneous medium is usually calculated by means of the following formula

$$Z_R = \frac{1}{\pi}\int_0^R \operatorname{argcosh} \frac{p(\xi)}{p(R)} d\xi \ , \tag{2}$$

where Z_R is the depth of the turning point of the ray that emerges on the surface at epicentral distance R, $p(\xi) = dt/d\xi$ is the derivative of the travel-time curve with respect to the epicentral distance. The velocity at the turning point was determined as the reciprocal derivative of the travel-time curve at the point of observation, $v(Z_R) = 1/p(R)$. For details, we refer the reader to the papers by Janský and Junge (1966), and Novotný et al. (2004).

The smoothed data, i.e. the smoothed travel times t and corresponding epicentral distances R and ξ, were inserted into formula (2) to obtain the velocity-depth function. The resultant curve is displayed in Fig. 3 and denoted by number 1. The vertical cross-section of the P-wave velocity is characterized by relatively low superficial velocities (lower than v_P = 4.5 km/s) which are followed by higher velocities up to a velocity round v_P = 5.8 km/s at a depth of about 1 km.

To estimate the accuracy of the smoothed 1-D model, the method of delete-one jackknifing was used (Tichelaar and Ruff, 1989), i.e. interpretations were repeatedly performed for data in which one travel-time point had been successively omitted. Two of these models that most deviated from the mean curve 1 are shown in Fig. 3 as the dashed lines.

Results of laboratory tests

The ultrasonic v_P velocity was measured under laboratory conditions among others physical and mechanical properties of selected rock samples. Three cylindrical cores were prepared from greywacke blocks taken in the quarry Jakubčovice nad Odrou. Their rough dimensions were 60 x 30 mm, so that the requisite slederness ratio (2:1) should be kept. These specimens underwent the measurement of ultrasonic v_P velocities with confining pressure up to 150 MPa in the laboratory of the Department of the Physical Properties of Rocks of the Geological Institute AS CR in Prague. Using three specimens, three sets of velocity measurements with the 10 MPa step of compressive strength were obtained. A simplified course of the investigated function is displayed in Fig. 2, i.e. only averaged values of all three and/or two sets of ultrasonic v_P velocities are considered.

In the course of laboratory experiments only specimens No. 1 and No. 2 enabled us to investigate ultrasonic velocity within the whole span of compressive strength applied, i.e. 0 – 150 MPa, whilst the specimen No. 3 made it possible to undergo the experiment only up to maximum compression of 110 MPa. When seeking the best fitting approximation for the averaged P-wave velocity vs. compressive strength, the following functions were applied:

- polynomial of the 3-rd order (cubic function) $v_P = a_0 + a_1 p + a_2 p^2 + a_3 p^3$, (3)
- rational function $v_P = \dfrac{a_0 + a_1 p + a_2 p^2}{1 + b_1 p}$, (4)

where v_P is the velocity of P-waves in m/s and p is the compressive strength in MPa.

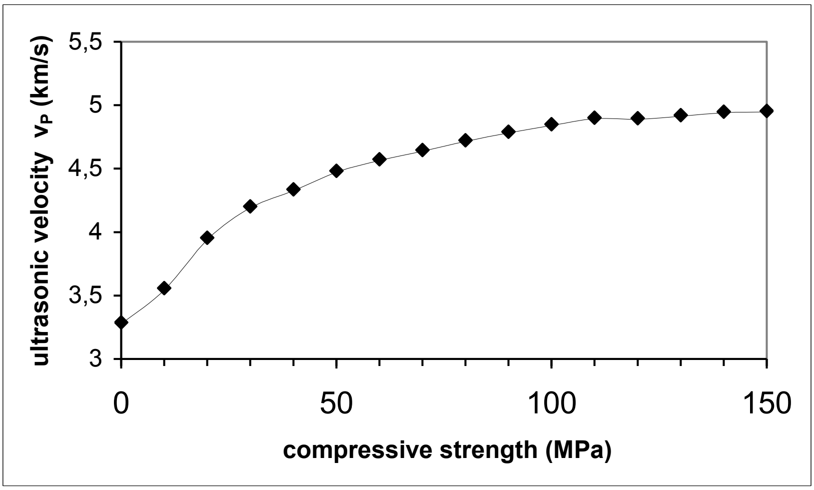

Fig.2 Dependence of averaged values of ultrasonic v_P velocity versus compressive strength applied (according to Holub et al., 2006).

Three values of velocities v_P for every discrete value of compression strength up to 110 MPa and two velocity values for specimens corresponding to a pressure of 120-150 MPa were considered. From the point of view of the root mean square deviations (RMS), the rational and cubic polynomial functions gave almost the same results. The overview of calculated values of all coefficients is given in Table 1.

Tab.1 Parameters for the polynomial (Eq. 3) and rational (Eq. 4) approximation of the dependence of v_P on compressive strength.

Sample No.	strength range (MPa)	a_0	a_1	a_2	a_3	b_1	RMS (m/s)	Equation
1+2+3	0-110	3248	42.41	-0.4451	0.001816	-	27	(3)
1+2+3	0-150	2187	36.56	-0.2957	0.00085	-	83	(3)
1+2+3	0-150	3256	91.64	-0.06032	-	0.0149	78	(4)

Figure 3 represents the velocity-depth function along the profile from the quarry Jakubčovice nad Odrou to the seismic station OKC (curve 1). The dependence of the ultrasonic velocity v_P vs. compressive strength (curve 2) was approximated with rational function (4) defined by the parameters from the last row of Table 1.

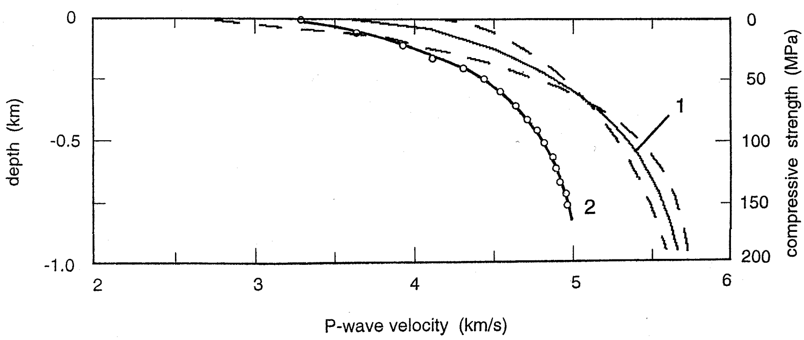

Fig.3 Approximation curves from seismic field measurements (curve 1) and from laboratory tests using compression strengths of 0-150 MPa (curve 2). The dashed lines show the most extreme variations of the velocity model obtained by the method of delete-one jackknifing.

The laboratory measurements yield a small value of surface velocity, even less than 3.5 km/s. Moreover, with increasing compressive strength, ultrasonic velocities in greywacke remain significantly lower than velocities measured *in-situ*. This velocity difference probably indicates that the rocks with increasing depth qualitatively differ from the greywacke, which creates the uppermost part of the geological profile.

It should be pointed out that the mentioned difference between the refraction and laboratory measurements in northern Moravia is remarkable, and has not been observed in other parts of the Bohemian Massif yet. For example, the refraction measurements across the Smrčiny-Fichtelgebirge pluton in the western Bohemian Massif (Málek et al., 2004) agreed with laboratory measurements rather well (Martínková et al., 2000). However, the analysis of these problems extends beyond the scope of the present paper, and should be discussed elsewhere. We mainly wanted to approximate some experimental seismic curves.

Conclusions

Two basic data sets considered in the present paper were obtained from previous field seismic measurements and from laboratory measurements on rock samples. The present study was aimed at searching a reasonable smoothing procedure of both data sets. Two approaches in smoothing were tested, namely the rational and polynomial approximations that are described by Eqs. (1), (3) and (4).

The considered travel-time curve of refracted P-waves was fitted better by the rational function defined by Eq. (1) than by a polynomial. Opposite to it the changes of seismic velocities due to increasing compressive strength approximated by the rational as well as by polynomial functions, given by Eqs. (3) and (4), yielded comparable results.

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